Table 3 ΔT_{cycle} for quasisteady-state contacts

Specimen type	τ	Bi	ΔT_{cycle}, °C
Brass	0.10	3.21	3.50
Brass	0.20	1.92	3.99
Brass	0.20	3.52	4.78
Aluminum	0.39	0.98	4.93
Brass	0.40	3.58	6.27
Brass	0.50	3.77	5.50
Copper	0.55	0.98	5.89
Copper	0.55	1.07	6.05

temperature is affected more radically than the contact temperature by the contact and separation time. These results are in agreement with the results of Vick and Ozisik.[4]

Also apparent from Fig. 3 and Ref. 4 is the dependence of the interface temperature "envelope":

$$\Delta T_{cycle} = T_{interface,separation} - T_{interface,contact} \qquad (1)$$

on τ. Less obvious, but discernible from the experimental data in Table 3, is the apparent dependence of ΔT_{cycle} on Bi. For each case of similar values of τ, larger values of ΔT_{cycle} correspond to the larger values of Bi.

Comparison of the experimental results with those of Howard[5] requires computation of three additional terms (l_c, the bar length equivalent to $\frac{1}{2}h_c$ for steady-state contact; l_i, the bar length equivalent to the resistance due to periodic interruption of the heat flow; and l_T, the bar length equivalent to the combined effects of contact resistance and periodic interruption of the heat flow) and measurement of the temperature distribution in each specimen for a fixed contact, steady-state condition. This additional measurement was made in conjunction with the periodic measurements for a set of brass and a set of copper test specimens. The quantity l_c is determined by extrapolation of a linear curve fit of the temperature distribution at steady state in the single contact measurement. The quantity l_T is determined from the periodic contact experiment by averaging the temperatures measured at each location for the entire cycle—both contact and separation—and extrapolating a linear curve fit of this time-averaged temperature distribution. The quantity l_i is then computed from the definition[1,3,5] $2l_i = l_T - 2l_c$. The results are nondimensionalized into the form of Howard and Sutton[1,3,5] by squaring the length parameter, multiplying by the cycle frequency, f, and dividing by the thermal diffusivity. This manipulation gives values of $f(l_i)^2/\alpha = 0.1592$ and $f(l_c)^2/\alpha = 0.0043$ for brass and $f(l_i)^2/\alpha = 0.3841$ and $f(l_c)^2/\alpha = 0.1192$ for copper in the current experimental results. Using the parameters ft_c and $f(l_i)^2/\alpha$, the results of Howard and Sutton[3,5] are interpolated to give $f(l_c)^2/\alpha$ of 0.0033 for the brass and 0.114 for the copper specimens.

Conclusions

Experimental observations of the heat-transfer and temperature distributions across periodically contacting surfaces are presented for low contact pressure, moderate interface temperature, identical materials across the contact interface, and equal contact and separation times in any cycle. From the results, it is seen that, for fixed values of the thermal contact conductance, changes in t_c (indicated by changes in τ) change the temperature distribution at the end of the separation portion of the cycle. For these cases, changes in τ do not significantly alter the temperature distribution during specimen contact. For fixed cycle contact/separation times, changes in h_c (indicated by changes in Bi) influence both the position and magnitude of the envelope of temperatures attained in the quasi-steady-state condition. Finally, over the range of the experiments, there is good agreement in the form and actual results with Vick and Ozisik,[4] Howard and Sutton,[3] and Howard[5]—indicating the ability of these models to accurately predict interactions for the quasisteady periodic contact phenomena.

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Mixed Convection on a Horizontal Surface with Injection or Suction

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Introduction

THERMAL buoyancy forces can strongly influence forced convection flow and heat transfer at relatively small flow velocities and for large temperature differences between the surface and external flow.[1,2] It is also well known that injection or withdrawal of fluid through the surface, as in mass transfer cooling, can significantly modify the flowfield and affect the rate of heat transfer in forced or free convection.[3] This Note is concerned with the combined effect of buoyancy and surface mass transfer on forced convection flows. Specifically, the problem of mixed convection on a horizontal plate with uniform suction or blowing from the surface is considered. The relative effects of buoyancy and surface mass transfer on the wall friction and heat transfer rates are analyzed using a composite measure of these two nonsimilar effects.

Analysis

Consider steady-state laminar flow over a semi-infinite horizontal flat plate maintained at constant temperature T_W. The external flow consists of a uniform freestream with velocity U_∞ and temperature T_∞. The buoyancy force associated with the temperature difference $\Delta T = T_W - T_\infty$ induces a streamwise favorable pressure gradient that interacts with the boundary layer. The fluid properties are constant except for the density variations contributing to the buoyancy force. The governing boundary-layer equations for the problem can be found elsewhere [see Eqs. (1-7) in Ref. 2]. For mass addition or removal through the surface, the y component of the velocity has the value

$$v = \pm V \qquad \text{at} \quad y = 0 \qquad (1)$$

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where the plus sign refers to injection (blowing) and the minus sign refers to withdrawal (suction) and V is the uniform blowing and or suction velocity. The conservation equations and boundary conditions can be transformed as follows:

$$F''' + \frac{1}{2}ff'' + \frac{1}{2}\xi\left(f''\frac{\partial f}{\partial \xi} - f'\frac{\partial f'}{\partial \xi}\right)$$

$$+\frac{1}{2}(1-w)\xi\left(\eta\theta + \phi + \xi\frac{\partial \phi}{\partial \xi}\right) = 0 \tag{2a}$$

$$\frac{1}{Pr}\theta'' + \frac{1}{2}f\theta' + \frac{1}{2}\xi\left(\theta'\frac{\partial f}{\partial \xi} - f'\frac{\partial \theta}{\partial \xi}\right) = 0 \tag{2b}$$

$$\phi' = -\theta \tag{2c}$$

$$f' = 0, \qquad f \pm w\xi = 0, \qquad \theta = 1 \qquad \text{at } \eta = 0 \tag{3a}$$

$$f' \to 1, \qquad \theta \to 0, \qquad \phi \to 0 \qquad \text{as } \eta \to \infty \tag{3b}$$

where the prime denotes differentiation with respect to the pseudosimilarity variable $\eta = Re_x^{1/2}y/x$ and Re_x represents the Reynolds number. In the preceding equations, $f(\xi,\eta) = \psi(x,y)/(\nu Re_x^{1/2})$ is the dimensionless stream function, $\theta(\xi,\eta) = [T(x,y) - T_\infty]/(T_W - T_\infty)$ is the dimensionless temperature, and $\phi(\xi,\eta) = \int_\eta^\infty \theta \, d\eta$ is an auxiliary variable. The nonsimilarity variable $\xi = Re_x^{-1/2}x/L$ is a dimensionless measure of the distance along the plate. The reference length L for the problem is representative of the combined effect of buoyancy and surface mass transfer:

$$L^{-1} = L_B^{-1} + L_M^{-1} \equiv g\beta\Delta T/U_\infty^2 + V/\nu \tag{4}$$

The terms on the right-hand side of Eq. (4) can be regarded as the typical inverse length scales for the buoyancy and the surface mass transfer effects, respectively. Similarly, the parameter w in Eqs. (2a) and (3a) is defined as

$$w = L_M^{-1}/(L_B^{-1} + L_M^{-1}) = (V/\nu)/(g\beta\Delta T/U_\infty^2 + V/\nu),$$
$$0 \le w \le 1 \tag{5}$$

and is the measure of the surface mass transfer effect compared to the combined buoyancy and mass transfer effect. [In this respect, L^{-1} and w are analogous to the extinction coefficient (inverse mean beam length) and the scattering albedo in radiative heat transfer through participating media.] If $V = 0$ ($w = 0$), then ξ reduces to the nonsimilarity variable for mixed convection on an impermeable horizontal plate with $\xi = Gr_x/Re_x^{5/2}$. On the other hand, ξ represents the nonsimilarity variable for flow over a plate with uniform blowing or suction with $\xi = (V/U_\infty)\, Re_x^{1/2}$ if $\Delta T = 0$ ($w = 1$).

Results and Discussion

Equations (2) and (3) were solved numerically by a finite-difference collocation scheme described in Ref. 4. Solutions were carried out for Prandtl numbers of 0.7 and 7 and for w values ranging from 0 to 1 for both injection and withdrawal of fluid. The parameter ξ was varied from 0 to 10 (except, of course, in the limiting case $w = 1$ with injection). The solutions agreed very well with the results tabulated by Ramachandran et al.[5] for the special case of mixed convection on an impermeable plate ($w = 0$). Table 1 presents selected results for $f''(\xi,0)$ and $-\theta'(\xi,0)$, which are representative of the friction factor and the Nusselt number, respectively.

In the limiting case with $w = 1$, the effect of withdrawal (suction) on forced convection flow is to increase friction and heat transfer at the wall compared to those for an impermeable plate ($\xi = 0$). Injection of fluid (blowing) has the opposite effect. Addition of mass promotes S-shaped velocity profiles, which decreases skin friction and eventually leads to separation of the boundary layer. Separation, $(\partial u/\partial y)_w = 0$, for blowing ($w = 1$) occurs for a relatively small mass transfer rate, i.e., at a low ξ value (around $\xi = 0.7$–0.8). In the other

Table 1 Results for $A = f''(\xi,0)$ and $B = -\theta'(\xi,0)$

	w	$\xi = 0.2$		$\xi = 0.5$		$\xi = 1.0$		$\xi = 1.5$		$\xi = 2.0$	
		$Pr = 0.7$[a]									
		A	B	A	B	A	B	A	B	A	B
W[b]	1.0	0.462	0.386	0.679	0.537	1.093	0.817	1.545	1.119	2.018	1.438
W	0.8	0.486	0.375	0.696	0.497	1.038	1.710	1.382	0.938	1.733	1.179
W	0.6	0.510	0.365	0.724	0.460	1.031	0.616	1.309	0.776	1.571	0.943
W	0.4	0.535	0.355	0.758	0.428	1.058	0.535	1.312	0.639	1.539	0.743
W	0.2	0.559	0.345	0.797	0.398	1.105	0.466	1.361	0.525	1.586	0.580
	0.0	0.582	0.336	0.837	0.370	1.161	0.405	1.427	0.430	1.662	0.449
I[c]	0.2	0.524	0.315	0.726	0.326	0.984	0.328	1.195	0.323	1.379	0.315
I	0.4	0.462	0.292	0.607	0.281	0.797	0.255	0.925	0.228	1.084	0.202
I	0.6	0.393	0.268	0.476	0.234	0.596	0.185	0.696	0.144	0.780	0.111
I	0.8	0.316	0.241	0.323	0.181	0.368	0.114	0.416	0.071	0.459	0.043
I	1.0	0.218	0.207	0.083	0.095	—	—	—	—	—	—
		$Pr = 7.0$[d]									
		A	B	A	B	A	B	A	B	A	B
W	1.0	0.462	1.565	0.679	3.454	1.093	6.989	1.545	10.507	2.018	13.986
W	0.8	0.442	1.354	0.607	2.782	0.922	5.574	1.270	8.398	1.637	11.187
W	0.6	0.427	1.159	0.546	2.147	0.761	4.161	1.006	6.281	1.269	8.382
W	0.4	0.417	0.982	0.505	1.575	0.626	2.788	0.763	4.157	0.921	5.562
W	0.2	0.413	0.824	0.493	1.096	0.574	1.596	0.626	2.158	0.668	2.777
	0.0	0.414	0.686	0.512	0.727	0.642	0.776	0.752	0.812	0.849	0.842
I	0.2	0.388	0.552	0.481	0.437	0.648	0.297	0.818	0.200	0.987	0.133
I	0.4	0.358	0.432	0.438	0.233	0.610	0.082	0.784	0.027	0.943	0.008
I	0.6	0.322	0.326	0.375	0.103	0.514	0.014	0.643	0.002	0.749	0.000
I	0.8	0.277	0.233	0.279	0.031	0.349	0.001	0.411	0.000	0.458	0.000
I	1.0	0.218	0.154	0.083	0.002	—	—	—	—	—	—

[a] $f''(0,0) = 0.332$, $-\theta'(0,0) = 0.293$. [b] Withdrawal. [c] Injection. [d] $f''(0,0) = 0.332$, $-\theta'(0,0) = 0.646$.

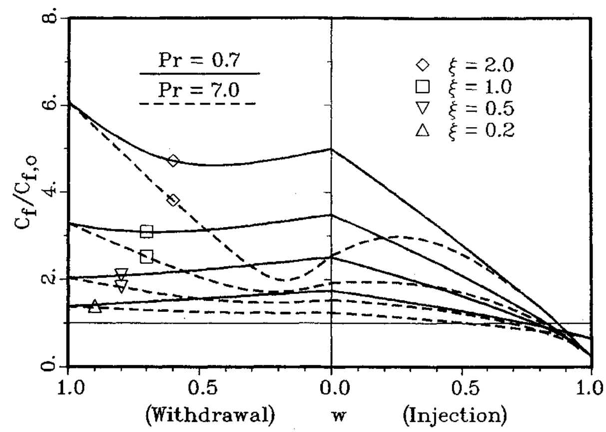

Fig. 1 Deviation of the local friction factor from the pure forced convection result.

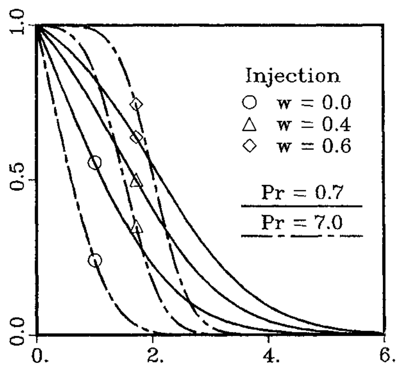

Fig. 2 Temperature profiles for various values of the parameter w in the injection case with $\xi = 2$.

limiting case with $w = 0$, which represents the mixed convection on an impermeable plate, the friction coefficient increases with increasing values of the buoyancy parameter ξ. Fluids with lower Prandtl numbers are more sensitive to the buoyancy parameter and may exhibit overshoots in velocity beyond the freestream value.[5]

When both thermal buoyancy and surface mass transfer effects are present ($0 < w < 1$), the wall friction and heat-transfer results vary between these limiting cases (from withdrawal to injection) for a given ξ. Figure 1 shows the deviation of the friction coefficient C_f from the pure forced convection result $C_{f,0}$ (the C_f value for $\xi = 0$). In general, withdrawal of fluid opposes the flow that is aided by buoyancy. This is due to the fact that withdrawal results in lower temperatures overall in the fluid, which in turn reduces the buoyancy effect. At lower values of ξ and/or w, the net effect of withdrawal is to reduce the overshooting tendency and slow down the flow. This leads to a decrease in friction at the plate. At higher ξ values, suction dominates momentum transfer in the boundary layer as w is increased. Vorticity is confined to a thin layer, resulting in a steeper velocity gradient and a higher shear stress at the wall. These trends are accentuated further for $Pr = 7$.

In the case of injection, the friction coefficient decreases with w, as blowing from the surface impedes the flow parallel to the plate. For $Pr = 7$, however, an initial increase in the friction coefficient is observed for higher ξ values as w is increased. This can be explained by referring to the corresponding temperature profiles presented in Fig. 2. For flow over an impermeable plate ($w = 0$), the thermal boundary layer is thinner for $Pr = 7$, with overall lower temperatures than those for $Pr = 0.7$. With injection of fluid ($0 < w < 1$), the temperature profiles become increasingly S shaped. For $Pr = 7$, the temperature variations are steeper, with the temperature drop confined to a thinner region. This results in a thicker "isothermal" buffer layer and higher fluid temperatures next to the wall. Thus, fluid in the vicinity of the plate is subject to a larger buoyancy-induced pressure gradient, causing it to accelerate and increase friction. For $Pr = 0.7$, the contribution to the buoyancy-induced pressure gradient by injection is relatively small. Hence, for small to moderate w values, injection of fluid decreases skin friction in fluids with small Pr but contributes to it in fluids with large Pr. With increasing w, the effect of buoyancy diminishes and corresponding injection limit ($w = 1$) is approached independently of Pr.

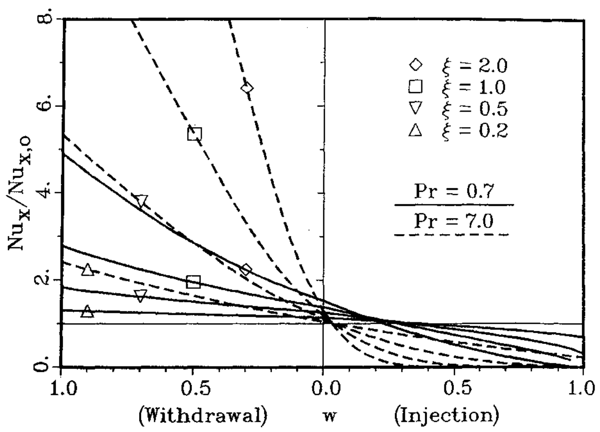

Fig. 3 Deviation of the local Nusselt number from the pure forced convection result.

The deviation of the local Nusselt number Nu_x from the forced convection result $Nu_{x,0}$ (the Nusselt number for $\xi = 0$) is shown in Fig. 3. As expected, these variations are monotonically decreasing from the withdrawal limit to the injection limit and are more intense for $Pr = 7$.

Conclusion

Mixed convection on a horizontal plate with uniform surface mass transfer is studied. To this end, a measure of the surface mass transfer effect relative to the combined buoyancy and mass transfer effects is introduced. For small to moderate mass transfer ratios, withdrawal of fluid opposes buoyancy-aided flow and decreases friction, whereas injection of fluid has the opposite effect for higher Prandtl numbers. These trends are reversed for increased mass transfer rates, as the effects of suction or blowing overcome buoyancy forces and dominate the flowfield. Overall, heat transfer in buoyancy-aided flows is increased by withdrawal and decreased by injection.

The weighted measuring used in this study to combine the two nonsimilar effects can be applied to other boundary-layer problems that contain more than one nonsimilarity parameter. The present analysis can be readily extended to surfaces with the uniform heat flux condition $q_w =$ constant (see the Appendix). Other problems that can be treated in this manner include mixed convection along a vertical plate with surface mass transfer, mixed convection about a vertical cylinder, and free convection about a vertical cylinder with surface mass transfer.

Appendix

The transformed system of equations for this problem under the uniform heat flux boundary condition is given by

$$f''' + \frac{1}{2}ff'' + \frac{1}{2}\xi\left(f''\frac{\partial f'}{\partial \xi} - f'\frac{\partial f'}{\partial \xi}\right)$$

$$+ (1-w)^2\xi^2\left(\frac{1}{2}\eta\theta + \phi + \frac{1}{2}\xi\frac{\partial \phi}{\partial \xi}\right) = 0 \qquad \text{(A1a)}$$

$$\frac{1}{Pr}\theta'' + \frac{1}{2}(f\theta' - f'\theta) + \frac{1}{2}\xi\left(\theta'\frac{\partial f}{\partial \xi} - f'\frac{\partial \theta}{\partial \xi}\right) = 0 \qquad \text{(A1b)}$$

$$\phi' = -\theta \qquad \text{(A1c)}$$

$$f' = 0, \quad f \pm w\xi = 0, \quad \theta' = -1 \quad \text{at} \quad \eta = 0 \qquad \text{(A2a)}$$

$$f' \rightarrow 1, \quad \theta \rightarrow 0 \quad \text{as} \quad \eta \rightarrow \infty \qquad \text{(A2b)}$$

where f, ξ, η, and ϕ are defined as before. The dimensionless temperature is defined as $\theta(\xi,\eta) = (T - T_\infty)Re_x^{1/2}/(q_w x/k)$. The reference length L is given by

$$L^{-2} = L_B^{-2} + L_M^{-2} \equiv g\beta q_w/kU_\infty^2 + V^2/\nu^2 \quad \text{with} \quad w = (V/\nu)L \qquad \text{(A3)}$$

When $q_w = 0$ ($w = 1$), the equations reduce to those for flow over a plate with uniform mass transfer, $\xi = (V/U_\infty)Re_x^{1/2}$. For $V = 0$ ($w = 0$), the equations represent those for mixed convection over a horizontal impermeable plate with $\xi = (Gr_x/Re_x^3)^{1/2} = \chi^{1/2}$, where χ is the buoyancy parameter used in Ref. 6 [note that $\chi(\partial/\partial\chi) = \frac{1}{2}\xi(\partial/\partial\xi)$].

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